# **Technical supplement**

#### Design of arch ribs - summary

The Hampton Court Bridge consists of two types of concrete structure: central piers which are primarily mass structures supported on piles and the arches which are framed structures. The former resist loads on them mainly due to the arrangement of their overall weight, whilst the arches resist loads and their self weight by careful configuration of their geometry.

In the arches made of reinforced concrete there are three force acting on its members:

Forces perpendicular to the direction of span (or its axis) called shear forces - V

Forces which act along its length (such as in columns or struts) called axial or normal forces - N

Bending actions called bending moments - M

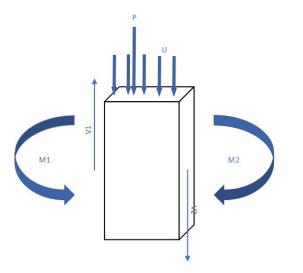
Members may also experience twisting moments (or torsion), but these would be considered negligible and therefore not taken into account in the calculations.

The application of any of the above forces, either singly or in combination will cause movement (deflections), however small. It is these movements that create stresses in the structural member experiencing the forces.

Reinforced concrete arches can experience all the above forces and are complicated to design. Rigorous methods were only being established in the early 1900's and built on earlier work in the mid 1800's by Navier, Castigliano and Rankine who had applied their theories to investigate the behaviour of masonry (voussoir) arches.

The simplest structure is a straight beam (or member) spanning between two supports at its ends. Under its own weight and with additional applied loads it will sag slightly. If it is free to rotate at its ends, it will sag more than if it were held tight at the end. The former is called simply supported and the latter, fixed (or encastré.) In practice, beams can span continuously over several supports which complicates how forces are distributed. At Hampton Court, the ends of the arches are fixed into the piers and each can be considered separately from the other, the two side spans being shorter than the central one.

For a beam, the forces shown diagrammatically below as a point load P and uniformly distributed load U create stresses internally at any section along the beam's length as shown.



Here M1 and M2 represent internal moment bending actions and V1 and V2 represent shear forces. All these must balance out and the material forming the structure, reinforced concrete at Hampton Court, must resist the stresses arising.

The calculation of the forces for a simple beam are straightforward although in a bridge many combinations of loading must be considered, both dead load (the weight of the finished structure) and live loads imposed by traffic. Calculations for the encastré arches at Hampton Court would have been very much more complicated as it is a statically indeterminate structure. It also has varying distributed dead loads due to the changing depths along its length and (in theory) varying point loads from the vertical column struts they supported. Although graphical methods were no doubt used, a taste of the theory available at the time can be seen from the mathematics' calculus and algebraic equations shown below to calculate axial forces, bending moments and ultimately shearing forces in the rib and the thrust on the piers and bridge abutments.

ELASTIC ARCHES AND RINGS 263 12.5. The segmental-arc cantilever .- Fig. 12.6 represents a segmental

cantilever rib, encastré at A and carrying a single vertical load P at  $\theta$  from the radius OC. The angle AOC is  $\phi$  and the rib carries at the free end C angle Ado Is  $\phi$  and the ino carries at the free the of a moment  $M_{\phi}$  a tangential force  $H_{\phi}$  and a radial force  $V_{\phi}$  acting as shown. These are independent actions and the displacements of C can be deter-

actions and this displacements of  $\circ$  of the distribution of the distribution of the set of the distribution of the distribut force are

 $\mathrm{M}{=}\mathrm{M}_{0}{-}\mathrm{H}_{0}\mathrm{R}(1{-}\mathrm{cos}\;\alpha){-}\mathrm{V}_{0}\mathrm{R}\;\mathrm{sin}\;\alpha{+}[\mathrm{PR}(\mathrm{sin}\;\alpha{-}\mathrm{sin}\;\theta)]\Big)$ (12.1) $T \!=\! H_0 \cos \alpha \!-\! V_0 \sin \alpha \!+\! [P \sin \alpha]. \label{eq:tau}$ and

The terms in P only occur between  $\alpha = \phi$  and  $\alpha = \theta$ ; the others between  $\alpha = \phi$  and  $\alpha = 0$ .

If the effects of radial shearing forces are neglected as being inappreciable, the angular displacement,  $\mu_0$ , and the component displacements,  $\hat{h}_0$  and  $v_0$ , of C are

$$\mu_0 {=} \frac{\partial \mathbf{U}}{\partial \mathbf{M_0}}, \ h_0 {=} \frac{\partial \mathbf{U}}{\partial \mathbf{H_0}} \ \text{and} \ v_0 {=} \frac{\partial \mathbf{U}}{\partial \mathbf{V_0}},$$

where U is the strain energy of the rib due to bending moments and tangential forces. Then

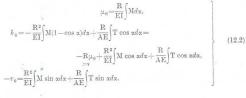
$$\begin{split} \mu_{0} &= \frac{1}{\mathrm{EI}} \Big[ \mathrm{M} \frac{\partial \mathrm{M}}{\partial \mathrm{M}_{0}} ds + \frac{1}{\mathrm{AE}} \Big] \mathrm{T} \frac{\partial \mathrm{T}}{\partial \mathrm{M}_{0}} ds, \\ h_{0} &= \frac{1}{\mathrm{EI}} \Big[ \mathrm{M} \frac{\partial \mathrm{M}}{\partial \mathrm{H}_{0}} ds + \frac{1}{\mathrm{AE}} \Big] \mathrm{T} \frac{\partial \mathrm{T}}{\partial \mathrm{H}_{0}} ds, \\ & 1 \int_{\mathrm{C}} \partial \mathrm{M} = 1 \int_{\mathrm{C}} \partial \mathrm{M} = 1 \int_{\mathrm{C}} \partial \mathrm{T} \end{split}$$

and  $v_0 = \frac{1}{\mathrm{EI}} \left[ \mathrm{M} \frac{\partial \mathrm{M}}{\partial \mathrm{V}_0} ds + \frac{1}{\mathrm{AE}} \right] \mathrm{T} \frac{\partial \mathrm{T}}{\partial \mathrm{V}_0} ds,$ where EI is the constant flexural rigidity of the rib and AE is its constant

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extensional rigidity. If the values of the derivatives for bending moment and tangential force

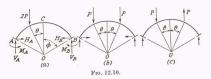
taken from equation (12.1) are substituted in the above they become



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12.6. The encastré segmental arch rib with a concentrated load.—The results of the last paragraph will now be used to determine the resultant actions in an encastré arch rib carrying concentrated loads acting in any directions at specified positions. All such loads can be resolved into vertical and horizontal components and so general solutions for a single vertical and horizontal load respectively enable the complete resultant actions to be determined by superposition.

horizontal load respectively characteristic the theory determined by superposition. Further, a single load can be represented, as shown in Fig. 12.10, by a pair of symmetrically disposed loads superposed on a pair of akew-sym-metrical loads. Thus the reactions at A and B for the loading shown at (a) can be found by superposing the reactions at these points due to the sym-metrical system (b) and the skew-symmetrical system (c).



Vertical load on arch rib. Considering first the symmetrical system (b) of Fig. 12.10, the resultant actions at C consist of a couple  $M_0$  and a thrust  $H_0$ , as shown in Fig. 12.6. From considerations of symmetry,  $V_0$  is zero and the only displacement of section C is in a vertical direction. Hence, the conditions at C are

$$V_0 = 0$$
;  $\mu_0 = 0$ ;  $h_0 = 0$ .

Substituting these conditions in equations (12.3) and (12.4), simultaneous equations are obtained to determine  $H_0$  and  $M_0$ . These are

 $\frac{\mathrm{M}_{0}\phi}{\mathrm{B}} - \mathrm{H}_{0}(\phi - \sin \phi) + \mathrm{P}(\cos \theta - \cos \phi - \theta' \sin \theta) = 0,$ 

 $\frac{4\beta M_0 \sin \phi}{2} + H_0 \{\beta (2\phi + \sin 2\phi - 4 \sin \phi) + 2\phi + \sin 2\phi\}$ R  $+P\{2\beta(\sin\phi-\sin\theta)^{2}+\cos 2\theta-\cos 2\phi\}=0,$ 

nd their solution gives  

$$\begin{array}{l} & \beta\{\phi(\cos 2\phi + \cos 2\theta - 2) + 4\sin\phi(\theta\sin\theta - \cos\phi + \cos\theta)\}\\ & -\phi(\cos 2\theta - \cos 2\phi)\\ \hline & 2p = \frac{1}{2\beta(2\phi^3 + \phi\sin 2\phi - 4\sin^2\phi) + 2\phi(2\phi + \sin 2\phi)},\\ & M_{\theta} = \frac{1}{\phi}[H_{\theta}(\phi - \sin\phi) - P(\cos\theta - \cos\phi - \theta'\sin\theta)]. \end{array} \right\}$$

$$\begin{array}{l} \\ \end{array}$$

$$\begin{array}{l} & 1 \\ 1(2.14) \\ \hline & M_{\theta} = \frac{1}{\phi}[H_{\theta}(\phi - \sin\phi) - P(\cos\theta - \cos\phi - \theta'\sin\theta)]. \end{array}$$

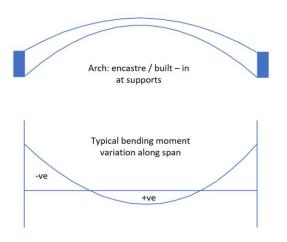
Under the skew-symmetrical loading of Fig. satisfied at C are 0;

$$M_0 = 0$$
;  $H_0 = 0$ ;  $v_0 = 0$ 

and so from equation (12.5),  $\beta [-V_0(2\phi - \sin 2\phi) + P(2\theta' - \sin 2\phi - \sin 2\theta + 4\sin \theta \cos \phi)]$  $-V_0(2\phi - \sin 2\phi) + P(2\theta' - \sin 2\phi + \sin 2\theta) = 0,$ 

Influence lines may also have been used to compute the variation of, for example, bending moments near the piers due to changes in vertical loading from the column struts. Interestingly, due to their encastré nature, more reinforcing bars were placed at the piers than at the crown centre of the arch rib. This may be because the bending action can be greater at the supports than at the mid span, although in reverse direction.



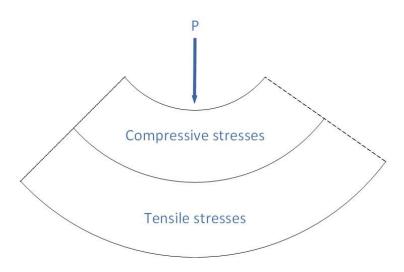


This does oversimplify the behaviour of the structure. Modern computerised methods can take into account the way in which loads would be distributed not just along but across the bridge deck and its supporting beams before transmission to the arches via the struts themselves.

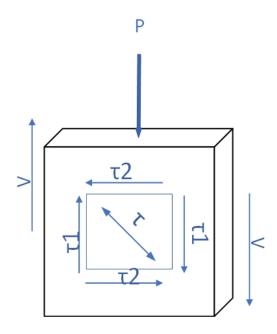
Another challenge is that the arch structure only has strength when it is finished. Consequently, the centring needed to use a prior arch structure albeit of lighter construction, leading to a significant part of the overall cost. At Hampton Court, prefabricated steel arches were used although designed as simply supported beams for relative simplicity and to eliminate horizontal thrusts at the supports.

The distribution of stresses due to the above forces at any section of even a simple beam can be extremely complicated. The following, however, provides a broad outline of what is happening and, consequently, why reinforcement is placed in concrete.

Firstly, stresses from the action of bending under its own weight or an applied load P arise as shown below. With a beam deflecting downwards due to such loading, the top section of the beam is in compression and experiencing compressive stresses. The bottom section is in tension and experiencing tensile stresses.



Secondly, shear forces at any point in the beam must balance out.



For the small element within a beam shown above, the applied forces V within it from the externally applied load P create the internal stresses  $\tau 1$  which are balanced by complementary stresses  $\tau 2$ . One of the resulting stresses  $\tau$  is another tensile one. This reaches a maximum at a certain orientation and is called a diagonal principal tensile stress.

Hence, tensile stresses arise from both bending and shear forces.

Stresses also arise due to movements in structures which occur in ways other than reversible deformations arising from applied loads. Under constant load, concrete suffers from permanent displacement due to creep. Thermal movement also occurs due to variations in atmospheric temperature overall and temperature differences over its surfaces. Finally, as concrete dries out it shrinks. Usefully, however, this last movement is the main contributor to the bond established between concrete and any reinforcement added enabling the strength in the steel to be mobilised.

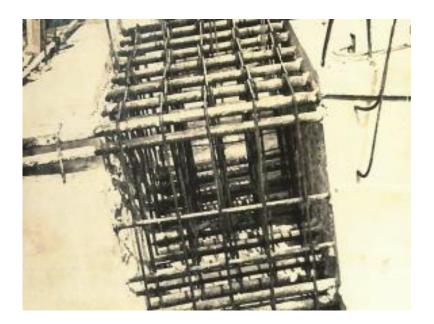
The creation and use of concrete has a long history. For example, a form of concrete, pozzolana, was used in Roman times which initially incorporated volcanic material around Pompeii. Being of plastic constituency, it was recognised that this material could be poured into three dimensional shapes and consequently became much cheaper to use than the cutting of stonework into precise shapes. In the UK, a significant advance was made in 1859 by using an artificial cement (Portland cement) for use, after a long series of tests, in the London main drainage works by engineer John Grant. In 1898, "Concrete Bob" McAlpine designed and built in mass (unreinforced) concrete the Borrodale Bridge and the Glenfinnan Viaduct for the West Highland Railway.

Concrete itself has to be designed both to be suitable for the environmental conditions under which it is placed and in order to achieve within the required timescale the desired strength and workability. For a concrete to be free from internal holes (voids) there has to be sufficient cement to fill the voids between the fine aggregate particles (sand) to make a dense mortar, and sufficient mortar to fill the voids between the coarse aggregate particles. The aggregates themselves are selected with regard to some nine different factors. The addition of water is also a fine art as too little creates an unworkable mixture and too much, over an optimum, reduces its strength. At Hampton Court, five different concrete mixes were employed. The problem with concrete is that although it is strong in compression, it is particularly weak in tension. Although there is variation in the stresses permitted, concrete at Hampton Court would probably have a working compressive strength of some 1,000 lb/sq in and crack in tension at around 300 lb/sq in, whereas its ultimate crushing strength in compression could be nearer ten times greater at 3,000 lb/sq in. Hence, concrete alone is useless for members subjected to bending since failure would occur at very low loads in regions where tension is developed. This limitation of the material can be overcome and the necessary resistance to tensile stresses can be provided by the insertion of steel rods at appropriate places. Rods can also be placed in the compression zone to add further strength in support of the concrete. Reinforcement used at Hampton Court is of mild steel and has a permissible working tensile strength of some 20,000 lb/sq in.

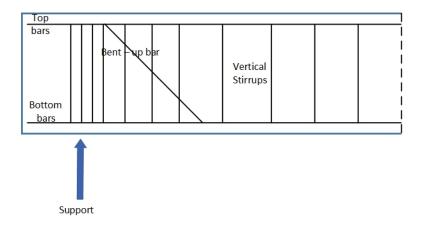
This composite structure is known as reinforced concrete and is made practicable by two fortunate circumstances. Steel and concrete have almost two identical coefficients of expansion, so that no serious internal stresses are set up by temperature changes. Further, when concrete sets in air it contracts and firmly grips reinforcement embedded within it. Consequently, loads are shared very effectively between the two materials and in such an ideal manner that design theory is greatly simplified.

### **Resisting stresses**

Any structure can have a combination of the three force actions described above. Within the arch ribs at Hampton Court Bridge we can see the arrangement of reinforcement at the sections still exposed. These will have been designed to resist bending actions and shear, and in all probability neglecting the effects of any torsion.



This shows main reinforcing bars at the top and bottom of the ribs to accommodate bending and any normal forces thrusting towards the piers. In total, 32 bars were placed at the encastré supports with 20 bars placed at the centre of the span. The vertical stirrups help not only to keep the rods in place before concreting but also to resist the vertical components of the principal tensile stresses due to shear. These are spaced more closely together when the forces are greater. It is also common practice to resist shear by bending up rods from the bottom layers to the top layers as appropriate, most particularly in beams as shown diagrammatically below.



Reinforcement, is also added to flat slabs to help control cracking due to shrinkage, and to struts and piles to further strengthen concrete when only under compression from normal forces.

To assist in the design of reinforced concrete structures, Codes of Practice were progressively developed. Approaches to the design have changed considerably since the first national Code of Practice for reinforced concrete was published in 1934 (D.S.I.R.) with a first revision in 1948 (CP114), and the first guidance for bridges issued by the Ministry of War Transport in 1945. Further significant advances include the practice of prestressing concrete. This aims to stress the members artificially to a state of compression, so that under conditions of superimposed loading, the normal tendency for tensile stress to develop only goes to nullifying the artificial compressive prestress. The first code of practice for prestressed concrete was published in 1959 (CP 115). The first unified code for the structural use of concrete was published in 1972 (CP110), although withdrawn in 1985 and its successor BS8110 published in 1997. The current standard in 2021 for bridges is Eurocode 2, introduced in April 2010.

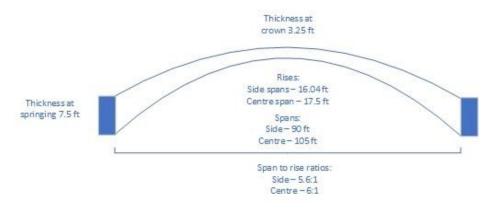
### Design of arch bridges – more detail

### Aesthetics

For a bridge to be aesthetically satisfactory the following conditions should be fulfilled. The design must suit the particular conditions existing at the site and the materials of the bridge must be appropriate for the adopted design. The bridge as a whole and in every part must look what it really is. The proportions, masses and lines of the bridge must be beautiful and the texture and colour of the materials must be pleasing. The ornament and architectural treatment of the bridge must be suited to its materials, while the bridge both in general design and colour must harmonize with its surroundings. Spandrel filled arches are not economical for large spans, so that when masonry or other facing is desired it is usual to employ false spandrel walls. This is somewhat deceptive, as the eye assumes the arch to be spandrel filled structurally. (Adapted from Chettoe and Adams.)

### Shape

Structurally, the most important factor affecting arch design is the span to rise ratio with a shape that is generally one of three types, segmental (or constant), parabolic, which is most often used, or elliptic. At Hampton Court the shape is elliptic with overall span characteristics as shown in the figure.



In arch bridges the most economical span to rise ratio for all types is generally considered to be between 3 to 1 and 6 to 1. Owing to the enormous thrusts set up and the difficulty of making the abutments entirely unyielding it is generally impracticable to construct arches with a span to rise ratio greater than 10 to 1 and this ratio is considered to give a very flat and uneconomical arch. At Hampton Court, the thickness at the springing is 2.3 times the thickness at the crown, slightly outside the normal range of 1.5 to 2.

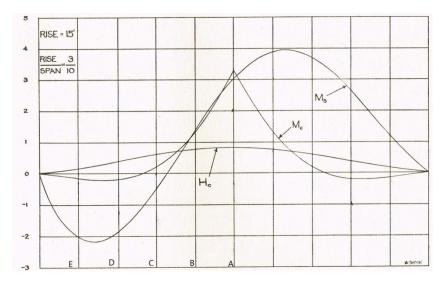
### **Influence of loading**

For many centuries structures of arch form have been in use made up of wedge-shaped blocks of masonry known as voussoirs, which were usually set in jointing material of mortar or cement, although this was not necessary for their stability. The shape of the arch and strength of the abutments alone determined its stability with the central aim of containing the dead load in the arch and ensuring that tensile stresses were avoided. Mathematical theories for evaluating structural behaviour were only rigorously being developed during the 19<sup>th</sup> century. Nevertheless, lessons from this time were carried over in assessing the behaviour of arches made of materials which possessed both compressive and tensile strength; structural steel (as in the 3<sup>rd</sup> Hampton Court Bridge) and the current one made of reinforced concrete. With all types of arch, moments due to dead load can be eliminated ("rib shortening" in the fixed arch apart) when the dead load thrust line coincides with the arch axis throughout. The rediscovery this century of Young's 19th century design method for this condition also demonstrated this theoretically.

### Influence lines

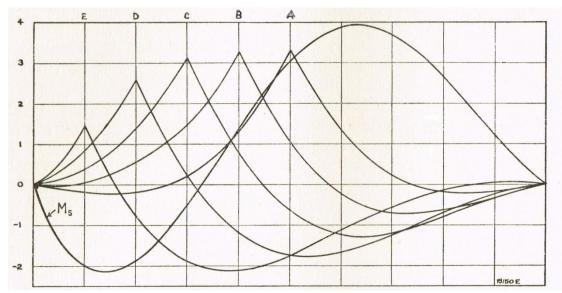
The simplest and safest way to analyse all types of arch was by employing influence lines, and these should ideally have been constructed from "first-principles" without making unnecessary or unreasonable assumptions. Each line for an influence line is drawn for one point only of the structure to give one of the selective forces (bending moment, shear etc) the value of that force at that point for different positions of the load. Compare that to a bending moment diagram, for example, which gives the bending moment at all points for one position of the load.

Although the geometric shape of the arch is not identical to that at Hampton Court, its influence line profiles could well have been similar to those shown in the figure below. These show the bending moment at the springings (Ms), bending moment (Mc) at the crown, A, and horizontal thrust (Hc) at the crown.

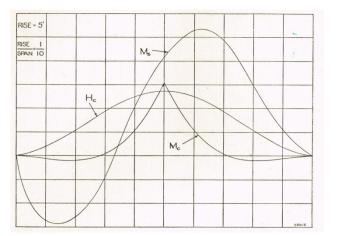


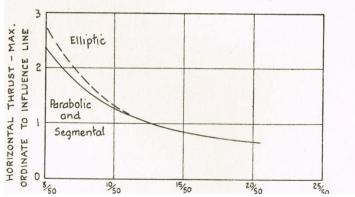
For example, both Mc and Ms change direction as the load traverses from the left hand side, through the centre of the span at the crown, to the right hand side of the span whilst the horizontal thrust at the crown remains positive throughout.

Once the dead load is determined and distributed along the entire span, it remains unchanged. The positioning of the live load, however, requires many options to be considered and the influence lines show where they should be placed to create the most severe bending moment cases, both positive and negative. Influence lines also need to be calculated for different positions along the span. For example, at locations A to E their profiles for the bending moment at the springing (Ms) follow the pattern shown in the following figure.



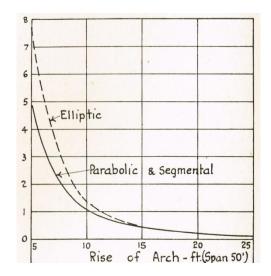
When arches become flatter, the horizontal thrust (Hc) at the crown begins to increase exponentially, which requires the abutments and intermediate piers to be progressively more resistant to any movements that this tends to induce.





### Thrust due to temperature changes

The temperature range usually considered in design was plus or minus 30 degrees F. Difficulties due to temperature changes, as well as shrinkage and rib shortening, mount rapidly as the arch becomes flatter. Consequently, a rise to span ratio of 1/10 could reasonably be taken as the low limit as shown in the left hand side of the figure below.



The thrusts on the abutments become so large that the difficulties are not confined to the arch ring itself. As the arch section is increased to cope with the increased stresses, this results in its moment of inertia increasing, and at approximately the cube of its depth. This draws additional internal moments towards it and the total stresses are further increased. Adverse temperature effects are also compounded. There comes the stage when an increase in section results in an unmanageable set of these fibre stresses under a combination of the most unfavorable circumstances. It therefore pays to keep the depths of the section down to a minimum, and to use a fairly large percentage of reinforcement.

Thus the variation in thickness of the arch ring is of importance. Stress requirements indicate that the springing thickness should be usually from 1.5 to 2 times the thickness at the crown. As the rise to span ratio increases, the required increase in springing thickness above that at the crown decreases, and with a large rise to span ratio, very little increase is required. For flat arches the 50% increase (1.5) is insufficient, and the springing thickness in some cases will need to be more than twice that at the crown. This is due to redistribution of internal moments as indicated above, and consequently its stiffness.

Neglecting to adequately mitigate the effect of temperature stresses, as at the relatively flat metal third bridge at Hampton Court, will eventually induce failure in the structural members.

### Analysis of the arches and determination of reinforcement

Clearly, the analysis of the structure needs to be rigorous. In the absence of anything but slide rules and electro mechanical computation machines available in the 1930's, systematic methods were essential. This required the mathematical stages to be broken down into manageable chunks.

An example of calculations which were potentially used for the arches at Hampton Court is shown in the figure.

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Readers may be able to recognise some of the entries and the equations are compatible with those provided in an earlier section. The point loads along the arch represent the loads imposed by columns supporting combinations of cross beams which, in turn, span across other columns. The beams need to be stiff compared with the columns and the bending moments in them could have been determined by the Theorem of Three Moments developed by Clapeyron in 1857.

### Deck expansion, arch deflection and column behaviour

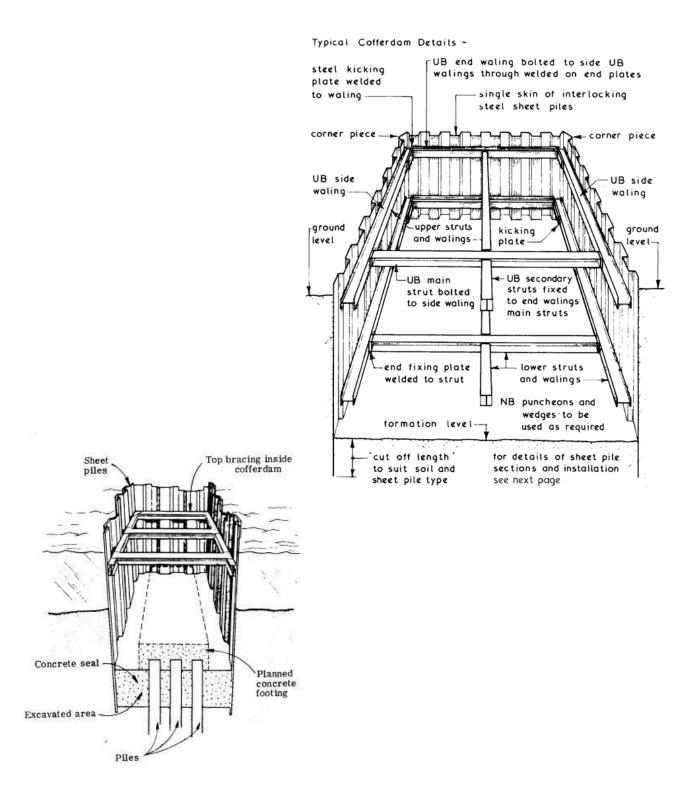
The arch ends are fixed so that, apart from any movement of the abutments, the span length cannot vary. But since the arch is flexible, temperature changes result in a rise or fall of the crown. Live load causes a similar movement. The deck itself should be free to expand and contract, and expansion joints are provided for this purpose over the substructure supports.

The elastic movement of the arch itself will be somewhat restrained by the columns and beams and by the monolithic construction with the deck at the crown, but expansion joints in the deck will limit this restraint. On the other hand, expansion and contraction of the deck between expansion joints will induce bending moments in the columns. Bending moments in these members are also developed as a result of deflections in the deck arising from the varying live load. The longer columns are also more flexible.

It is generally desirable to keep the direct stress in all the columns fairly low to allow for the secondary bending stresses which would not have been accurately calculated. It is also advisable to detail diagonal shear bars at both ends of the column as the maximum moment and shear at these locations occur together. The effect of these loading and stress interactions had been the subject of experimental research at the University of Illinois in 1931 and are too involved to simplify here.

### **Cofferdam structure**

Typical arrangements for cofferdams placed in water and also deep excavation in ground are shown below.



Soils impose forces, both locally on the walls of the cofferdam, and on the structure as a whole, thus adding to loads from hydrostatic and dynamic forces from the river. Local forces are a major component of the lateral forces on the sheet pile walls, causing bending in the sheet piling and walings, and axial compression in the horizontal struts.

Apart from resisting the horizontal forces acting on the sheet piles as previously described, an additional consideration was to prevent water seeping through the bottom of the excavation which the concrete seal prevents. In extreme cases, the loss of strength in a river bed could lead to spontaneous liquefaction, requiring other temporary measures which in modern times include dewatering or ground freezing.

To assess the behaviour of the river bed, it was agreed by Thames Conservancy in October 1927 that borings could be made in order to extract samples. The science of soil mechanics could then be used to test the material in order to assess its strength and likely behaviour under the conditions it would experience. For example, would it be strong enough to resist the forces from the bridge piers without using piling to take most or all of the loads imposed by the bridge?

There is no information on how soil mechanics was in fact used in the design of foundations for the Hampton Court Bridge. However, the science was well developed at the time and the following description indicates the knowledge then available.

In soil mechanics, a soil was (and is) regarded as material originally produced by the disintegration of rocky portions of the earth's crust. Through deposition, consolidation and settlement through still water, consolidation by pressure, or dispersal and grading by the action of moving water, the final product is the clay, sand, marl, gravel or other soil which forms or supports the works of civil engineering.

Ignoring organic deposits, the two main soil classifications used for analysis and use are sand and clay, each of which contains particles with their own detailed classification and size grading distribution. Their engineering properties and characteristics are further complicated by other factors including clay mineral properties, water content and associated liquid, plastic and shrinkage limits, voids ratio and porosity.

A combination of all these factors leads to the overall strength of a soil which is based on its resistance to shearing forces and its maximum resistance to shearing stresses. When this is exceeded, failure occurs usually taking the form of surfaces of slip. The law governing shear failure in soils, generally known as Coulomb's Law, was first put forward in 1773. This provides a simple relationship between shear strength, cohesion, applied stress and angle of friction at any given point. Using the principles of stress analysis applied to an element of material in concrete design above, Coulomb's Law can be depicted conveniently by Mohr's Circle of stress. This representation is used in laboratory tests called tri-axial soil tests which enable shear strengths to be experimentally determined for different conditions of drainage of a soil sample.

It is not known whether tri-axial tests were used for the Hampton Court bridge as it would have been considered "state of the art" at the time. An alternative method that was commonly used for assessing soil strength was the "probing bar" or, as it is currently known, "standard penetration test." Typically, a 2-in-diameter tube is driven about 18 in. into the ground below the bottom of the borehole and the number of blows required to cause 12 in. of penetration forms a guide to the resistance of the stratum, the first 6 in. of penetration being ignored. The results become relatively less reliable as the soil becomes more cohesive.

Typical allowable bearing pressures used for foundations at the time are shown in the following table.

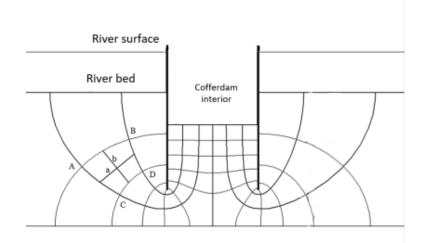
Foundation bed	Tons/sq.ft.
Moist sand, soft clay	1
Dry sand and clay mixed	2
Firm dry sand, firm clay	2-3
Firm gravel, coarse hard and compact sand, very hard clay, firm chalk	4-5
Shale in level beds, hard pan	6-8
Rock	8-100+

### Scour

Whenever the water level in a river rises, the river bed starts to move throughout the greater part of its length and width, and the bottom of the river goes down. This process is known as a scour and the failure of bridge piers due to this cause is not uncommon. Hence, the base of the foundation should be several feet below the level to which the river may scour during high water. The provision of piles mitigates this effect.

### Seepage under cofferdams

Seepage flow diagrams could have been prepared to estimate the direction, speed and quantity of water flow if a seal were not provided. In the diagram, which shows a typical arrangement of sheet piles, line AB joins up soil particles experiencing the same pressure (an equipotential line: as is CD) and AC and BD show two different pathways of elements of water (flow lines.)



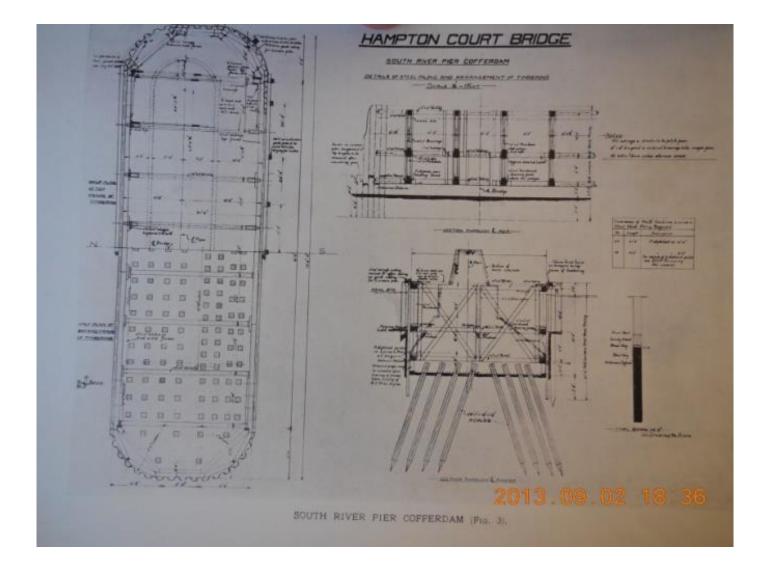
These diagrams are a graphical representation of an algebraic expression known as Laplace's equation. This applies to the flow of every incompressible fluid through an incompressible porous material of given permeability. The flow is also considered to be two dimensional. Although flow nets can be constructed graphically, they are commonly supplemented by

computational methods. This approach would have been known when Hampton Court Bridge was constructed, following the original work in hydraulics by the Austrian civil engineer Philipp Forchheimer who died in 1933.

The permeability of a soil is measured in the laboratory using a permeameter.

Again, it is not known whether any of the above fundamental soil mechanics techniques were used for the temporary and permanent designs at Hampton Court.

The actual arrangement for the south river pier cofferdam is shown below.



## Examples of major works by team members

Team member	Works
Sir Edwin Lutyens	British Ambassador's Residence, Washington D.C., U.S.A.
	British Medical Association, Tavistock Square, London
	Castle Drogo, Devon
	Cenotaph, Whitehall, London
	India Gate, Delhi, India
	Memorial to the Missing of the Somme, Thiepval, France
	Midland Bank Headquarters (former), Poultry, London
	Midland Bank, King Street, Manchester
	Reuter's Building, 85 Fleet Street, London
	Viceroy's House, now Rashtrapati Bhavan, Delhi, India
Holloway Brothers	Bridges
	1914 <u>Esk</u> Bridge Gretna, replacing one built by Thomas Telford in 1820
	1922-6 Bridge over the Thames at Reading
	1924-8 Royal Tweed Bridge at Berwick-upon-Tweed
	1930-3 Hampton Court Bridge
	1934-7 Chelsea Bridge The first self-anchored suspension bridge in the country and the first steel bridge built by Holloways
	1936-7 Towy Bridge at Carmarthen
	1936- King Ghazi and King Faisal Bridges across the Tigris at Bagdhad
	1936-40 Wandsworth Bridge
	1939- Bahrain swing bridge
	1945-50 Baghdad combined road-rail bridge across the Tigris
	1956-8 bridge over the Diyala River at Baqubah Iraq.
	Additionally: buildings, other civil engineering works, restoration of historic buildings, memorials and similar projects.
	https://en.wikipedia.org/wiki/Holloway_Brothers_(London)# Bridges
	accessed 5 <sup>th</sup> October 2020

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L G Mouchel	Bridges						
	Of 414 known reinforced concrete bridges built in the British						
	_						
	Isles between 1870 and 1914, Mouchel were responsible for						
	at least 300, commencing in 1901, of which 59 still exist.						
	Additionally, major structures including:						
	Royal Liver Building in Liverpool						
	London's Earls Court and Royal Victoria Dock						
	Football stands for Liverpool Football Club and Manchester						
	City Football Club						
	Cooling towers for London Battersea Power Station						
	The development of concrete bridges in the British Isles prior						
	to 1940, MM Chrimes, Proc. Instn Civ. Engrs Structs & Bldgs,						
	1996, 116, Aug./Nov., 404-431						
	And https://en.wikipedia.org/wiki/Mouchel						
	accessed 5th October 2020						